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PLANETARY OCCULTATIONS:

A review of the methods of prediction, the results of astrometric analysis and the future prospects

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Summary

The times of occultations of stars by the planets, including the minor planets, and the natural satellites are predicted since the observation of such events can provide valuable information about the sizes, shapes and atmospheres (if any) of these bodies. The astrometric results of recently-observed occultations are summarised and the results of searches for future occultations are listed. Two appendices give further details of occultations by Jupiter and Ganymede.

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PLANETARY OCCULTATIONS

History. The possibility of using planetary occultations to improve our knowledge of the planets has been appreciated for many years. A valuable summary of previous work on the subject, up to about 1923, is contained in a thesis by Dr. L.J. Comrie entitled "Planetary Occultations and their Observation". Comrie was the Superintendent of H.M. Nautical Almanac Office from 1930 to 1936. The only known copy of his thesis ~~known to the author~~ is a typewritten one in the library of this Office.

Part I: Prediction

The first stage in the process of prediction is to make a preliminary search to detect any close approaches (appulses) within stated limits. To enable this search to be performed efficiently, special astrometric ephemerides of the planets, Venus - Neptune, up to 1980 have been created on a magnetic tape. Other magnetic tapes have been created which contain star positions, referred to the equator and equinox of 1950.0, of all those stars in the SAO Star Catalog which could possibly be occulted by the particular planet concerned. A search program then compares a star tape with a planet tape and prints out details of any appulses within selected limits of declination. In the cases of Jupiter and Saturn, these limits are increased to allow for the possibility of occultations by the Galilean satellites and by Titan.

The list of appulses is then examined carefully and those which could possibly give rise to an occultation visible from the Earth's surface are given further consideration. At this stage it is possible to build up a mental picture of the circumstances of the occultation, including those areas which are in daylight, those which are below the horizon and the probable areas of visibility of the occultation itself. Another computer program is then used to calculate the times of the phases of the occultation as seen from selected observatories.

An additional step in the prediction process arises when there is the possibility of an occultation by a satellite, since an accurate ephemeris of the satellite relative to the planet is required.

For occultations by Pluto and Titan it is possible to observe stars fainter than those given in any star catalogue and different procedures are adopted. For these two bodies a series of photographs of the area of sky stars directly in the path are accurately measured and any close approaches investigated.

For radio sources the procedure is the same as that for the planets Venus - Neptune. The search for occultations by Mercury for stars down to $3^m.5$ only, and for occultations of infra-red sources, is done by hand since so few stars/sources are concerned. So far no planetary occultations of radio sources or infra-red sources have been observed.

It is of interest to note that, although this search for planetary occultations has been performed in the Nautical Almanac Office for the past 20 years, it is only since 1969 that a computer has been used for this purpose. Previously the comparison of the apparent ephemeris of the planet with star catalogues (using star positions corrected mentally for precession) was done by hand - an extremely laborious and time-consuming process.

Finally, the predictions are printed yearly in the Handbook of the British Astronomical Association and the more important ones also appear in the IAU Circulars.

Because of the uncertainties in the ephemerides of the planets and in the positions of the stars it is usually desirable to refine the predictions shortly before the predicted occultation occurs. This is an essential requirement for bodies of small angular size, such as the satellites and minor planets since a shift in declination of a few tenths of a second of arc may easily make all the difference between a nearly central occultation and no occultation at all. Such a refinement is also necessary for very slow moving bodies since a small shift in right ascension can then have a large effect on the predicted time. Ideally, it is best to take a photograph as soon as it is possible to get both objects on the same plate, but this is usually a counsel of perfection since the interval of time left before the occultation is likely to be less than that required to develop and measure the plates and to revise and circulate the improved prediction.

Part II: Analysis

Timed observations made from several different observing stations are combined in a solution to determine a value of the semi-diameter of the occulting body and its position relative to the star. The rate and direction of motion of the occulting body relative to the star are normally known

The equations of condition may be written in the form

$$\Delta\alpha \cos\delta \sin P + \Delta\delta \cos P + \Delta s = \Delta\sigma$$

where P is the position angle measured from the north point, $\Delta\sigma$ is the difference of the observed minus the computed semi-diameter for each observation, and $\Delta\alpha$, $\Delta\delta$, Δs are the corrections to the adopted ephemeris and angular semi-diameter of the occulting body. The quantities $\Delta\alpha$, $\Delta\delta$, Δs, $\Delta\sigma$ are in seconds of arc.

If the apparent disk of the occulting body were not circular but elliptical, the equations of condition should include additional terms for the determination of the eccentricity of the ellipse and the direction of its minor axis, assuming that the observational data are adequate for this purpose.

In making the analysis, it is necessary to decide on the weight to be assigned to each observation. It is customary to weight each observation inversely as the uncertainty in the timing and directly as the cosine of the angle between the radius and the path of the star relative to the occulting body. However, it is unwise to use this method if the occulting body has no atmosphere and may have irregularities on its surface which could cause variations in time of a similar size to the quoted uncertainty.

Several least-squares solutions are made, using a standard computer program with iteration until no further improvements are obtained. The program incorporates the correction due to the relativistic deflection of light in a gravitational field. Although negligible for satellites, it affects the observed radius of the planets - for Jupiter by over 50 kilometres.

Part III: Results

Observable occultations are extremely rare occurrences and the number of occultations observed in the last 20 years, from which it has been possible to derive positional and geometrical data, is only five.

However, on these occasions it has been possible to improve our knowledge of the diameters of the occulting bodies and of their position relative to the occulted star by about 2 orders of magnitude compared with more conventional methods, as will be seen from the results given below.

(a) Occultation of Regulus ($1^{m}.3$) by Venus ($-4^{m}.1$), 1959 July 7. This occultation was observed in daylight from South America, Europe, Africa and western Asia and, after sunset, from southern Asia. A total of 157 visual observations was used, covering both hemispheres of Venus, and an "occultation" diameter of 12 334 ± 4 km was obtained. There was no evidence of any oblateness.

6 167 ± 2

(b) Occultation of B.D. −17° 4388 ($7^{m}.8$) by Neptune ($7^{m}.7$), 1968 April 7 Photoelectric observations of both phases were made at several observatories in Australia, New Zealand and Japan. Using 13 observations, values for the equatorial diameter were found as follows:

Light-level	Diameter
1.0	50,940 ± (s.e.) 140 km
0.5	50,550 ± (s.e.) 80 km

where the radius for 0.5 light has been increased by 50 km for the scale height and 54 km for the relativistic deflection of light. This was the first occasion on which sufficient photoelectric observations were available for analysis to determine the diameter of any body other than the Moon. I have recently had the opportunity of re-assessing the relative weights of the observations at 0.5 light and find that, using the same 13 observations (of which 2 were rejected) and adopting various values for the flattening, the best fit (smallest standard error) is obtained by using a flattening of 0.023, resulting in a value for the equatorial diameter of 50570 ± (s.e.) 40 km. The polar diameter is 49420 km. The corresponding angular semi-diameters at 1 A.U. are $34''.86$ and $34''.07$, both ± $0''.03$. Assuming a mass of 1.0285 ± 0.0015 .10^{29}g, the density is 1.554 ± 0.002 g/cc, and the escape velocity 23.29 ± 0.02 km/s. For a standard magnitude of −7.06, the geometric albedo is 0.45. The time of conjunction in R.A. is 1968 April 7^{d} 16^{h} 19^{m} 02^{s} ± 1^{s} when the difference in declination (centre of planet minus star) was $+0''.494$ ± $0''.000$.

(c) Occultations of β Scorpii A ($2^{m}.9$) and C ($5^{m}.2$) by Jupiter ($-2^{m}.1$), 1971 May 13. These occultations were widely observed from Australia, New Zealand, Asia, Indonesia, South Africa and Eastern Europe. The two brighter components of this multiple system were separated by about 14" and in such a position angle that the Jovicentric latitudes sampled by star A were from S.47° to S. 70°, while those by C were S. 7° to S. 14°. The much greater range for A is accounted for by the fact that A was 2½ magnitudes brighter than C and was visible for a much longer time during its passage behind the

Any analysis which combined the observations of the two stars clearly required a knowledge of the separation and position angle. In particular it should be noted that any attempt to determine the oblateness of Jupiter will be critically dependent on the difference in declination of the two stars. Thus, the position of the star C relative to A was measured photographically at the U.S. Naval Observatory near the date of the occultation as follows:

$$\left.\begin{matrix} +0^{s}.3537 \\ +12''.697 \end{matrix}\right\} \pm 0''.008$$

This is the value used in their analysis by Hubbard and van Flandern (1972) and is equivalent to a separation of 13''.644 in a position angle of 21°.47. In the preliminary solutions made for the analysis given in Appendix 1 I discovered that all the disappearance observations of A had negative residuals and the reappearance observations had positive residuals, with the reverse occurring for the observations of C. This indicated some systematic effect and, since it was realised that the solution would be improved if the position angle was altered, an increase of 0°.10 was adopted, so that the values of $\Delta\alpha$ and $\Delta\delta$ actually used were:

$$\begin{matrix} +0^{s}.3552 \\ +12''.689 \end{matrix}$$

The analysis is given in the form of a paper in Appendix 1.

(d) <u>Occultation of β Scorpii C ($5^{m}.2$) by Io (5^{m}), 1971 May 14</u>. Six photoelectric observations of this event were used to determine the equatorial diameter as 3659 ± 4 km, but this value was partly dependent on the correction to the orbital longitude of Io. When this analysis was made, it was believed that Io was ahead of its ephemeris position by $1^{m}.2$. However, it is possible to use the positional results derived from the Jupiter analysis to determine the difference in R.A. between Io and Jupiter at the time of conjunction of Io with the star C. This value is $+8^{s}.8545$, whereas the value used for the previous Io analysis (Taylor, 1972) was $+8^{s}.8563$. Adoption of the new value indicates a revised time correction to the orbit of Io of $-0^{m}.9$ instead of $-1^{m}.2$. A revised difference in declination of $-27''.207$ is found, which differs by only $0''.031$ from the value used previously.

Is this significant? Certainly the third decimal is meaningless as the ephemeris of Jupiter in declination is only given to two decimals. In addition it is very doubtful if the ephemeris of Io relative to Jupiter is valid to an accuracy of $0''.01$, even though this value is actually going through a maximum during the occultation by Io. Another relevant factor is that it is only necessary to decrease the adopted flattening for Jupiter by 0.0013 to remove the discrepancy completely. Thus it can reasonably argued that the discrepancy is not significant. It may be remarked here that if such a value for the flattening (0.0579) is adopted the equatorial radius is found to be 71917 ± 4 km.

The effect of the revised time correction to the ephemeris of Io is quite small - the derived value for the diameter is reduced by 3.2 km, while the time of conjunction is $0^s.5$ earlier.

(e) Occultation of SAO 186800 ($8^m.0$) by Ganymede (5^m), 1972 June 7. Photoelectric observations of this occultation were made at Kavalur, India and Lembang, Java and the results of the analysis are given in Appendix B.

It is of interest to note that the only other known occultation of a star by Ganymede occurred on 1911 August 13, having been predicted by Banachiewicz[4]. Ristenpart's[5] analysis of the observations was based on an ephemeris of the satellite derived from photographic observations of the relative position of the two bodies before and after the occultation. The differences in R.A. and Dec. had quoted probable errors of $0''.10$ and $0''.06$, respectively, while the probable errors in the motions in these coordinates were $0''.003$ and $0''.002$ per minute respectively. From the analysis of the 1972 occultation we can calculate that the apparent semi-diameter was then $0''.66$, though the value derived by Ristenpart was about 30% greater.

It may be noted that Ristenpart had to use a few observations over a relatively short period of time in order to produce an ephemeris of the satellite relative to the star. Nowadays the theory of the motion of the satellites of Jupiter is such that an adequate ephemeris of a satellite can be calculated; and even though there may be small errors in the actual position, these can be detected and corrected in the analysis; the derived motion over the period of the occultation is known to a precision which is more than adequate for the analysis.

However, the observational material was extremely poor. The observers, mostly with small instruments with a resolving power of about 1", had to watch the satellite approaching a star about 2 magnitudes fainter than itself. The two images must have merged for some time before the occultation and remained merged for some time afterwards. At the instant of occultation the change in brightness of the combined image would have been only $0^m.27$ and almost impossible to detect visually. As a consequence, it would appear that the observed times of the changes in light were more likely to have been records of magnitude of Ganymede was then thought to be appreciably brighter than it is now and the predicted magnitude change was $0^m.6$, though with a proviso that it might be only $0^m.4$. In fact, using the diameter of Ganymede as 5270 km, the maximum duration of the 1911 occultation would have been about $3^m.5$. Yet five out of the six durations used in the analysis were larger than this value! Banachiewicz predicted a maximum duration of about $3^m.7$, assuming a diameter of about 5700 km.

The actual observed times are so discordant that no sensible solution can be obtained by using each time separately. It is only possible to derive any meaningful value for the radius by using the durations only (and assuming the times of both phases are in error by the same amount and that the errors are different for each station!). Using this method, Ristenpart obtained values for the equatorial and polar radii of 3754 and 3433 km respectively and derived a value for the flattening of 0.086, a value higher than for Jupiter itself and almost as much as that for Saturn. In the current analysis the revised value for the radius is 2635 km, about 1000 km less than that obtained in 1911.

Nowadays it is recommended that visual observers should use as small a telescope aperture as possible, consistent with obtaining a clear image of the star, so that the merging of the images occurs well before any change in brightness due to an occultation and observers are warned to record the change in brightness only. No attempt is made to enlist the support of visual observers unless the star is almost as bright as, or brighter than, the occulting body.

(f) Occultations by minor planets. Although predictions of such events have been issued for many years, only two observations have been made. An occultation of B.D. +6° 808 by Juno on 1958 February 19 was observed visually from Malmo, Sweden, and indicated that Juno's diameter was greater than 110 km. An occultation of B.D. -5° 5863 by Pallas on 1961 October 2 was observed photoelectrically from Naini Tal, India, and indicated that the diameter of Pallas was greater than 430 km.

Part IV: Future Prospects

No particularly outstanding occultations are predicted during the next few years, apart from the occultation of ϵ Geminorum by Mars on 1976 April 8. The following list gives details of occultations in the next year or so.

Planet or satellite	Star's limiting magnitude	Date	Star's visual magnitude	Search complete to end of year
Mercury	3.5	None		1975
Venus	7.5	1975 Dec. 13	(6.3)	1975
Mars	8.9	1975 Aug. 28 1975 Dec. 23 1976 Apr. 8	(8.3) (8.9) (3.2)	1976
Jupiter	8.9	1975 Feb. 14 1975 June 25	(7.5) (8.8)	1975
Saturn (+ Rings)	9.0	1974 Aug. 29 1975 Jan. 11	(9.0) (9.0)	1975
Uranus	9.0	1977 Mar. 10	(8.8)	1980
Neptune	9.0	1976 Nov. 15 1980 Nov. 24	(8.7) (8.5)	1980
Pluto	9.0	None		1980
Ceres, Pallas, Juno, Vesta	9.0	None		1975
Io, Europa, Ganymede, Callisto	8.0	None		1980
Rhea	9.0	1974 Aug. 29	(9.0)	1975

Plates of the areas ahead of Pluto and Saturn are currently being taken at the Royal Greenwich Observatory in order to predicteoccultations of stars down to 17^m for Pluto and 11^m for Titan and Iapetus. In addition, occultations of SAO stars by some of the other satellites of Saturn are also predicted. With the cooperation of the Institute of Theoretical Astronomy in Leningrad, which is supplying accurate ephemerides of selected minor planets, occultations by these bodies are also being predicted, though not very far in advance.

APPENDIX 1

Astrometric Analysis of the Occultation of β Scorpii by Jupiter on 1971 May 13

SUMMARY

An analysis of the timings of the occultation of β Scorpii by Jupiter on 1971 May 13 gives the value 71900 ± 2 km for the equatorial radius and 0.0592 for the flattening, at the half-intensity level in the atmosphere of Jupiter. Corrections to the ephemeris position of Jupiter and hence to the results of the earlier analysis of the occultation of β Scorpii by Io on 1971 May 14, are also obtained.

1. DATA AND METHODS OF ANALYSIS

The occultations of the components of β Scorpii A and C by Jupiter on 1971 May 13 were observed photoelectrically at six stations and the times at half-intensity level can be analysed to provide estimates of the size, shape and position of Jupiter. A similar analysis of the observations from Johannesburg and Naini Tal has already been made by Hubbard and Van Flandern (1972), and Lecacheux, Combes and Vapillon have given an analysis of their observations from Pretoria (1973). We have discussed all available data and have made an independent solution taking into account various other factors. The circumstances were such that the observed occultations occurred in high southern latitudes for component A and just south of the equator for component C.

Table 1 gives the observational data on which the analysis is based, together with the residuals from the final solution. The apparent paths of the stars behind Jupiter as seen from two observing stations widely separated in latitude, as deduced from the analysis, are given in Fig. 1.

TABLE I

Occultation Timings and final residuals

Obs. No.	Place	Phase	U.T. at half intensity		Weight	Residuals time	Residuals km	Position Angle	Jovicentric latitude
	Component A		h m s	s		s		°	°
1	Canberra	D	18 25 10	±2	2.5	−0.4	− 3.3	223	−58
2	"	R	19 39 24	±1	0	+2.4	+18.7	159	−57
3	Bickley	D	18 26 36.3	±1	0	−1.8	−15.6	224	−57
4	Naini Tal	D	18 23 01.6	±0.5	40	+0.3	+ 3.3	232	−49
5	" "	R	19 55 34.2	±0.5	40	0.0	+ 0.1	149	−47
6	Johannesburg	D	18 36 44.4	±0.5	40	−0.4	− 2.7	221	−60
7	"	R	19 48 47.1	±0.5	40	+0.1	+ 0.4	159	−57
8	Pretoria	D	18 36 42	±2	0	−1.6	−13.0	221	−60
9	"	R	19 48 50	±2	0	−2.7	−20.8	159	−57
10	Hartbeesport	D	18 36 38	±1	0	+4.5	+35.4	221	−60

Obs. No.	Place	Phase	U.T. at half intensity		Weight	Residuals time	Residuals km	Position Angle	Jovicentric latitude
	Component C		h m s			s		°	°
12	Canberra	D	17 43 26		0	+3.6	+57.4	268	−13
13	Naini Tal	R	20 12 03.6	±0.5	40	+0.2	+ 2.5	108	− 7
14	Johannesburg	D	17 53 11.4	±0.5	40	0.0	− 0.7	267	−14
15	"	R	20 14 23.4	±0.5	40	−0.2	− 2.8	113	−12
16	Pretoria	D	17 53 16	±3	0	−4.0	−63.0	267	−14
17	"	R	20 14 22	±2	0	−2.8	−43.9	113	−12

The positions of the observatories are given in the Astronomical Ephemeris for 1973 except for Johannesburg, for which the position used was: longitude $-1^h\ 52^m\ 17^s\!.93$; latitude $-26^\circ\ 11'\ 02''$; height 1790m. It is clear that the height of the station above sea level should not be ignored as it is of the same order of size as the standard error of a solution.

The ephemeris of Jupiter (right ascension, declination, tilt of axis, position angle of axis) was taken directly from the Astronomical Ephemeris using $\Delta T = +40^s$. The tabulated distance was corrected for light-time.

The FK4 position of β Scorpii A for 1950.0, together with its centennial proper motion (μ) was used. The values were:

$$\text{R.A.} \quad 16^h\ 02^m\ 31^s\!.507 \qquad \mu\alpha\ -0^s\!.041$$
$$\text{Dec.} \quad -19^\circ\ 40'\ 12''\!.45 \qquad \mu\delta\ -2''\!.13$$

The position of the star C relative to A has been measured photographically at the U.S. Naval Observatory near the date of occultation as follows:

$$\left.\begin{array}{ll}\Delta\alpha & +0^s\!.3537\\ \Delta\delta & +12''\!.697\end{array}\right\} \pm 0''\!.008$$

These values were used for preliminary solutions. However, since almost all the disappearance observations of A had negative residuals and the reappearances had positive residuals, with the reverse occurring for observations of C, it was realised that the solution would be improved if the position angle of C relative to A was increased. An increase of 0°.1 was adopted so that the values of $\Delta\alpha$ and $\Delta\delta$ actually used were:

$$\Delta\alpha = +0^s\!.3552$$
$$\Delta\delta = +12''\!.689$$

It is noted that the alterations correspond to $2\frac{1}{2}$ and 1 standard errors respectively in the photographically measured values of $\Delta\alpha$ and $\Delta\delta$.

There are a number of unknowns for which solutions could be made. These include two for the correction of the position of Jupiter relative to β Scorpii A, another two for the differences in R.A. and Dec. between the two stars A and C, and at least two more for the size and shape of Jupiter itself.

A study of the geometry of the occultation shows immediately that the observations of the star A occurred in high southern latitudes of Jupiter, while those of C occurred just south of its equator. In particular, it is noted that there are very few of the latter and, in fact, in the final solutions only three of them are retained. Thus it can be said that any solution for the shape and size of the planet, and also for the difference of position between the stars A and C, will be critically dependent on these three observations, particularly on the one observation of the disappearance of C. Thus it was decided to solve for three unknowns - the corrections to the adopted ephemeris of Jupiter in R.A. and Dec., and a correction to the adopted semi-diameter and the procedure adopted was the same as that described by Taylor (1972).

Since the apparent disk of Jupiter is elliptical, it would be desirable to determine the oblateness from the observations though, as explained above, an extra unknown (or unknowns) were not introduced into the equation of condition. Instead, various values of the flattening were used in the analysis.

It is considered that the timings of the occultation were not affected by the Red Spot.

The deflection of light in a gravitational field is generally accepted, and is large enough to be considered in this analysis. Owing to the oblateness of Jupiter, the effect will vary with latitude. The difference in the correction for those latitudes encountered by star A and those by star C is greater than the standard error of a solution. Therefore, each observation has been corrected for this effect individually when reducing the observations.

In order to allow for the effect of refraction, the scale height of the atmosphere should be added to the radius (Hubbard & Van Flandern, 1972). Following these authors, the radii determined from the observations have been corrected by the use of an equation which had the effect of adding about 30 km to the radii for observations of the star C and about 25 km to those for A (i.e. the correction was a function of latitude).

Following standard practice, each observation has been weighted inversely as the uncertainty in the timing.

5. RESULTS OF ANALYSIS

Several least squares solutions, using a standard computer program with iteration until no further improvements were obtained, were carried out in an attempt to determine whether the observational material was adequate to improve the assumed oblateness of Jupiter. It was soon found that the best solution, defined as that which gave the minimum standard error in the radius, was obtained using an oblateness of 0.0592, though within a range of oblateness of ±0.0010 the standard error only increased by about a kilometre.

The final solution gave the corrections to the right ascension and declination of Jupiter as $+0^{s}.0004$ (± (s.e.) $0^{s}.0000$ and $+0''.143$ ± (s.e.) $0''.001$. The equatorial radius was determined as 71899.7 ± (s.e.) 2.1 km. The corresponding polar radius was 67642.2 km. The corresponding angular semi-diameters at a distance of 1 A.U. are:

Equatorial	$99''.133$	$\pm 0''.003$
Polar	93.263	

For comparison a flattening of 0.0589 was also used giving an equatorial radius of 71904.4 ± (s.e.) 2.3 km. It was found that an increase in the flattening of 0.001 gave a decrease in the radius of about 14 km.

The differences between these values and those determined by Hubbard and Van Flandern are probably due to a combination of three factors taken into account in this paper: (a) the revision of the relative position of the stars; (b) the allowance for the height of the observing station; and (c) the variation in the relativity effect with latitude.

For comparison some previous values, using other methods, are given in the table below:

Table II

Investigator(s)	Damoiseau Pickering Sampson	Dollfus et al.	Hubbard & Van Flandern	Lecacheux Combes Vapillon	Taylor
Method	eclipse	micrometer	occultation	occultation	occultation
Radius at cloud top	71410	70870	71880	71350	71600
Flattening	-	0.061	0.060	(0.060)*	0.059

*adopted

Table I gives the residuals for each observation in terms of kilometres along the radius and also in time. The consistency of the seven observations stated to be $\pm 0^{s}.5$ are remarkably good as the largest residual is only $0^{s}.3$. Most of the other observations were not used in the final solution as their residuals were more than three times the standard error.

Although Jupiter is believed not to be in hydrostatic equilibrium, an attempt was made to see how closely the observations would fit such a model using an equation of the form

$$r = s\,(1 + \epsilon_2 P_2 \sin \varphi)$$

$$\text{where} \quad P_2 = \tfrac{1}{2}\,(3 \sin^2 \varphi - 1)$$

and r is the radius, s the mean radius, φ the latitude and ϵ_2 a measure of the oblateness.

The result was surprising. Other investigators (Hubbard & Van Flandern, 1972) have suggested that a reasonable value for ϵ_2 is about 0.041 but this analysis shows that the observations give the best fit to a value of ϵ_2 of about 0.021, a mean radius of about 72250 ± (s.e.) 11 km, and an impossibly large correction to the declination of Jupiter of $+1''.15$. Since it is known from the analysis of the observations of Io that the correction to the declination of Jupiter cannot be more than about $0''.1$, this result can be rejected immediately. There is one important lesson which can be drawn from this exercise, however. If no other considerations are available, a result which gave an s.e. of 11 km, corresponding to a timing error of about 1 second, is still plausible. Now an equatorial diameter of 71899.7 ± (s.e.) 2.1 km is also very plausible - but is it correct? As has been pointed out earlier, there are few accurate observations available and this exercise merely corroborates an obvious deduction - that the formal standard errors quoted in such an analysis are almost certainly gross underestimates of the true errors.

The time of conjunction with the star A in right ascension is found to be 1971 May $13^{d}\ 19^{h}\ 18^{m}\ 37^{s}.6 \pm 0^{s}.1$, with the difference in declination, planet-star, as $+18''.096 \pm 0''.000$.

Assuming a mass of 1.9×10^{30}g, the density is found to be 1.30 and the equatorial escape velocity 59.4 km/sec. The geometric albedo is 0.32 if the standard magnitude (at 1 A.U. from both Sun and Earth) is $-8^{m}.99$.

8. CONCLUSIONS

The rare opportunity afforded by the occultation by Jupiter of the two components of a double star has been used to improve our knowledge of the size and oblateness of the planet. It is surprising to find in the analysis of the observations, which had a jovicentric range of latitude of -47° to -60° for star A and -7° to -14° for star C, that an ellipsoidal figure of revolution can be found which satisfies the observations with a standard error of only 2 km. Because of the high correlation of the difference in declination of the two stars with the flattening, this standard error cannot be considered as realistic. It may be concluded that for the half intensity layer the equatorial radius almost certainly lies in the range 71880 - 71920 km, with a corresponding range in flattening from 0.061 to 0.058.

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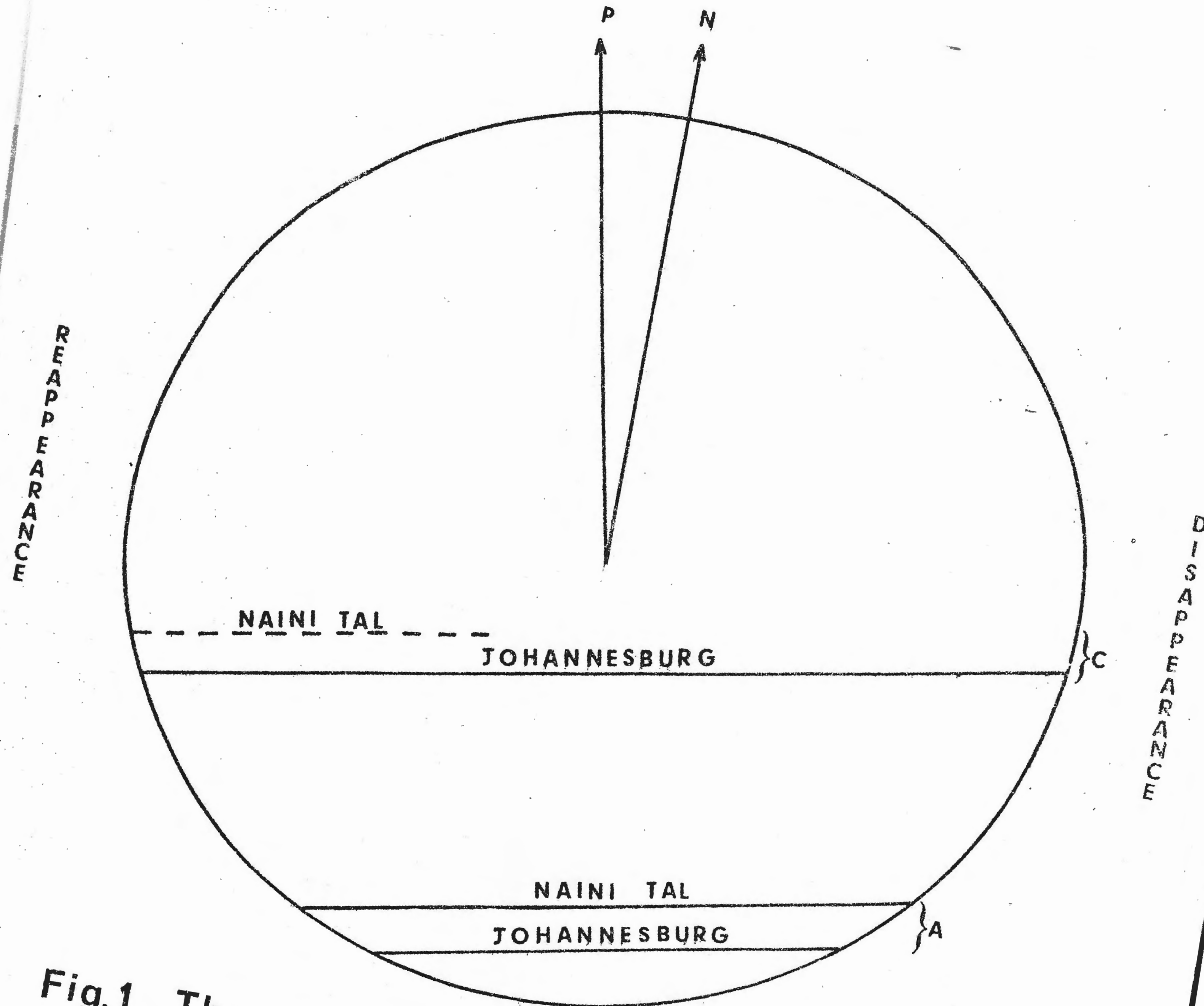

Fig. 1 The occultation of β Scorpii A and C by Jupiter on 1971 May 13 as seen from the observatories at Naini Tal and Johannesburg.

APPENDIX 2

The size of Ganymede determined from an occultation of a star

1. INTRODUCTION

The search for occultations of stars by planets carried out in H.M. Nautical Almanac Office indicated that the star SAO 186800 ($8^m.0$, KO) would be occulted by Jupiter on 1972 June 7. Further investigation indicated that the satellite III Jupiter, Ganymede, would also occult the star on the same day.

Since the largest single source of error in the predictions was likely to lie in the position of the star, photographs were taken from the Observatories at the Cape of Good Hope, South Africa and Perth, Australia. The plates were taken in March when Jupiter and Ganymede were moving eastwards past the star which they were to occult three months later when they were moving westwards. The relative positions of Ganymede and the star were measured and the predictions were refined as a result.

The predicted area of visibility was south-west Asia, northern Australia and east Africa and detailed predictions were issued on IAU Circular No. 2401. The uncertainty in the relative positions of the two bodies was then estimated as $0''.3$. The value adopted for the diameter was that given by Dollfus (1970), namely 5550 ± 130 km.
R = 2775

Ganymede is much brighter than the star and the expected change in intensity at the occultation was only about 5%. Nevertheless successful observations were made at Kavalur in India and from Lembang in Java, where an expedition from the U.S.A. had been sent to monitor the event. A second expedition from the U.S.A. to Darwin, Australia, was unsuccessful; the star and the limb of the satellite were separated by only $0''.02$ at their closest approach. The error in the prediction of the southern limit of the occultation was due to the combination of the errors in the adopted relative positions of the star and Ganymede and in the adopted diameter of Ganymede. The southern limit of the actual occultation passed about 50 km north of Darwin. The apparent paths of the star behind Ganymede as seen from the observing stations are shown in Fig. 1.

2. DATA

The positions of the observing stations and the times of observation are given in Table 1. Unfortunately there is a discrepancy between the times recorded at Kavalur and those recorded at Lembang. It would appear that there is a clock error of 5 seconds at one of these stations, though an investigation has failed to locate the error. The discrepancy was noticed as soon as an analysis was made for the determination of the diameter using the

Theoretically, if the positions of the star, Jupiter, and of Ganymede relative to Jupiter, were known sufficiently accurately it would be possible to decide at which station the error occurred. In practice, it is impossible as the corrections to the path of Ganymede obtained from (1) a solution using Lembang times $+5^{s}.0$ and (2) a solution using Kavalur times $-5^{s}.0$, differ by only $0^{s}.0030$ in RA and $0''.002$ in Declination. This difference is too small to be detectable observationally.

TABLE I

Observational Data

Station	Position	Height	Phase	Time (U.T.)	
		m		h m s	s
Kavalur, India	E. $5^{h}\ 15^{m}\ 18^{s}$ N. $12^{\circ}\ 33'\ 31''$	800	D R	18 49 21.2 18 52 41.2	±0.1 ±0.1
Lembang, Java	E. $7^{h}\ 10^{m}\ 27^{s}.84$ S. $6^{\circ}\ 49'\ 32''.9$	1300	D R	18 47 40.3 18 50 21.8	±1.2 ±1.2

The 1950.0 position of the star was taken as R.A. $18^{h}\ 22^{m}\ 41^{s}.833$, Dec. $-23^{\circ}\ 06'\ 37''.22$ and, assuming no proper motion, the apparent position at the time of conjunction in R.A. was found to be R.A. $18^{h}\ 24^{m}\ 05^{s}.9564$, Dec. $-23^{\circ}\ 05'\ 54''.014$, though in the actual analysis the apparent position of the star was calculated for each observation.

The ephemeris of Jupiter was derived from that given in the Astronomical Ephemeris using a value of $\Delta T = +42^{s}$. Thus, at $18^{h}\ 50^{m}$ U.T., the position of Jupiter was taken as R.A. $18^{h}\ 24^{m}\ 09^{s}.961$, Dec. $-23^{\circ}\ 06'\ 01''.99$ with motions per minute of $-0^{s}.02077$, $-0''.0169$ respectively.

The ephemeris of Ganymede relative to Jupiter used in the analysis was kindly supplied by Dr. D.T. Vu of the Bureau des Longitudes, Paris. The motion of the satellite was so uniform during the occultation that even had there been an error in orbital longitude corresponding to its motion in as much as 5 minutes of time there would have been no appreciable effect on this analysis. The values for the differences in R.A. and Dec., in the sense Ganymede - Jupiter, at $18^{h}\ 50^{m}$ U.T., were $-4^{s}.01200$, $+9''.0535$ and the motions per minute $-0^{s}.01511$, $-0''.0130$ respectively.

The distance of Ganymede dfrom the Earth was taken as 4.26414 A.U.

3. METHOD

The method of analysis was the same as that used for the occultation by Io on 1971 May 14 (Taylor 1972), whereby the equations of condition contain three unknowns - corrections to the right ascension, declination and semi-diameter of the occulting body. Since only four observations were available it is impossible to determine any oblateness of Ganymede and a simple circular cross-section has been assumed. It is noted that there are no reasons for assuming any detectable departure from a sphere.

Because of the discrepancy between the times of observation from the two stations, mentioned previously, two solutions were made, one by applying $-5^{s}.0$ to the times at Kavalur, the other by applying $+5^{s}.0$ to the times at Lembang. Apart from the time of conjunction, the results are almost identical as may be seen from Table II. All the observations were given equal weight.

The values of the density and escape velocity were made assuming a mass of $1.5525 \pm 0.0190\ 10^{6}$ g and the geometric albedo has been calculated using a standard magnitude of -1.62.

4. RESULTS

Residuals from these solutions, and other quantities, are given in Table II, and the results of the analysis are given afterwards.

TABLE II

Residuals

Station	Phase	Residuals* (a)	(b)	(c)	Position Angle	Latitude
		s	s	s	o	o
Kavalur	D	+2.04	+0.02	+0.02	275.8	+ 9.0
	R	+1.93	+0.02	+0.02	77.2	+ 9.6
Lembang	D	-3.07	-0.03	-0.02	229.6	-37.2
	R	-3.03	-0.03	-0.02	123.8	-37.0

(a) original times
(b) $-5^{s}.0$ applied to times at Kavalur
(c) $+5^{s}.0$ " " " " "

The corrections to the position of Ganymede are considerably smaller than the uncertainty in either its position or that of the star.

Diameter	5271 km
Density	2.03 ±0.03 g/cm^3
Escape Velocity	2.80 ±0.02 km/sec
Geometric Albedo	27%
Semi-diameter at 1 A.U.	3".634
Difference in declination at time of conjunction (planet minus star)	+1".060 ±0.000

	(b)	(c)
U.T. of Conjunction	$18^h\ 49^m\ 41^s.9$ ±$0^s.0$	$18^h\ 49^m\ 46^s.2$ ±$0^s.0$

For comparison the values of various quantities for Io determined from its occultation of β Scorpii C on 1971 May 13 are as follows:-

Density	2.82 ±0.34 g/cm^3
Escape Velocity	2.30 ±0.14 km/sec
Geometric Albedo	37%

5. DISCUSSION

The value given for the radius in this analysis is effectively an upper limit since there is still considerable uncertainty as to the density and extent of the atmosphere. Previous work has shown that the surface pressure is greater than 10^{-3} mb. If the pressure is as high as 1 mb the radius is reduced by about 140 km assuming reasonable values of the mean molecular weight and temperature.

The formal standard error in the diameter is only 1 km, but this is clearly unrealistic as will be seen from a comparison of the time residuals with the observer's quoted probable errors. It is more realistic to consider the standard error to be 20 - 30 km. Even this is a big improvement on previous older values. Previously various micrometer measures led to values for the radius ranging between 2210 and 3225 km.

ACKNOWLEDGMENTS

The success of this operation is due to the cooperation of many people - J. Churms and T. Russo at the Cape of Good Hope Observatory and B.J. Harris at the Perth Observatory for taking the preliminary photographs, and to J.C. Bhattacharyya, B. Hidijat, R. Carlson, T. Johnson, B.A. Smith and S.A. Smith for making the observations of the occultation.

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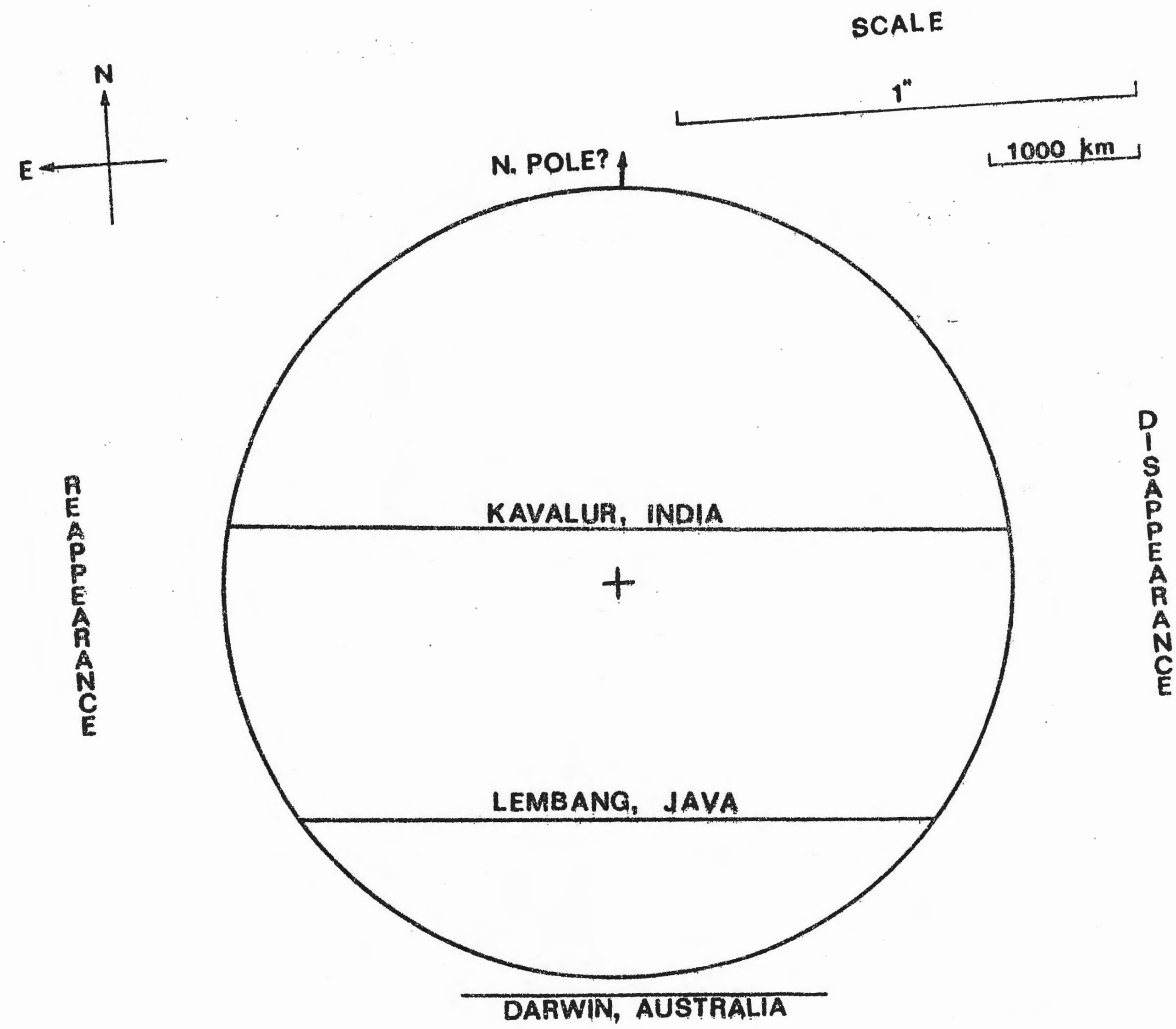

Fig.1 The apparent path of the star SAO 186800 behind Ganymede as seen from India and Java on 1972 June 7.

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